



Blatt 7

Lösung 7.1

$$\begin{aligned}
 \text{z.Z.:} \quad & \hat{\underline{K}}_I = (1 - \frac{1}{I}) \left[\hat{\underline{K}}_{I-1} + \frac{1}{I} (\vec{c}_I - \vec{\mu}_{I-1})(\vec{c}_I - \vec{\mu}_{I-1})^\top \right] \\
 = & \frac{1}{I} \sum_{i=1}^{I-1} (\vec{c}_i - \vec{\mu}_{I-1})(\vec{c}_i - \vec{\mu}_{I-1})^\top + \frac{1}{I} (\vec{c}_I - \vec{\mu}_{I-1})(\vec{c}_I - \vec{\mu}_{I-1})^\top - \frac{1}{I^2} (\vec{c}_I - \vec{\mu}_{I-1})(\vec{c}_I - \vec{\mu}_{I-1})^\top \\
 = & \frac{1}{I} \sum_{i=1}^I (\vec{c}_i - \vec{\mu}_{I-1})(\vec{c}_i - \vec{\mu}_{I-1})^\top - \frac{1}{(I-1)^2} \left(\frac{I-1}{I} (\vec{c}_I - \vec{\mu}_{I-1}) \frac{I-1}{I} (\vec{c}_I - \vec{\mu}_{I-1})^\top \right) \\
 & \text{mit } \vec{\mu}_I = (1 - \frac{1}{I})\vec{\mu}_{I-1} + \frac{1}{I}\vec{c}_I \Leftrightarrow \vec{\mu}_{I-1} = \frac{1}{I-1} (I\vec{\mu}_I - \vec{c}_I) \\
 = & \frac{1}{I} \sum_{i=1}^I (\vec{c}_i - \frac{I}{I-1}\vec{\mu}_I + \frac{1}{I-1}\vec{c}_I)(\vec{c}_i - \frac{I}{I-1}\vec{\mu}_I + \frac{1}{I-1}\vec{c}_I)^\top \\
 & - \frac{1}{(I-1)^2} \left(\frac{I-1}{I} (\vec{c}_I - \frac{1}{I-1} (I\vec{\mu}_I - \vec{c}_I)) \left(\frac{I-1}{I} (\vec{c}_I - \frac{1}{I-1} (I\vec{\mu}_I - \vec{c}_I)) \right)^\top \right) \\
 = & \frac{1}{I} \sum_{i=1}^I \left((\vec{c}_i - \vec{\mu}_I) + \frac{1}{I-1} (\vec{c}_I - \vec{\mu}_I) \right) \left((\vec{c}_i - \vec{\mu}_I) + \frac{1}{I-1} (\vec{c}_I - \vec{\mu}_I) \right)^\top \\
 & - \frac{1}{(I-1)^2} (\vec{c}_I - \vec{\mu}_I) (\vec{c}_I - \vec{\mu}_I)^\top \\
 = & \hat{\underline{K}}_I + \frac{1}{I} \sum_{i=1}^I (\vec{c}_i - \vec{\mu}_I) \left[\left(\frac{1}{I-1} \right) (\vec{c}_I - \vec{\mu}_I) \right]^\top + \frac{1}{I} \sum_{i=1}^I \left[\left(\frac{1}{I-1} \right) (\vec{c}_I - \vec{\mu}_I) \right] (\vec{c}_i - \vec{\mu}_I)^\top \\
 & + \frac{1}{I} \sum_{i=1}^I \left[\left(\frac{1}{I-1} \right) (\vec{c}_I - \vec{\mu}_I) \right] \left[\left(\frac{1}{I-1} \right) (\vec{c}_I - \vec{\mu}_I) \right]^\top - \frac{1}{(I-1)^2} (\vec{c}_I - \vec{\mu}_I) (\vec{c}_I - \vec{\mu}_I)^\top \\
 = & \hat{\underline{K}}_I + \left(\sum_{i=1}^I \vec{c}_i - I\vec{\mu}_I \right) [\dots]^\top + [\dots] \left(\sum_{i=1}^I \vec{c}_i - I\vec{\mu}_I \right)^\top \\
 & + \left(\frac{1}{(I-1)^2} (\vec{c}_I - \vec{\mu}_I) (\vec{c}_I - \vec{\mu}_I)^\top \right) \frac{1}{I} \sum_{i=1}^I 1 - \frac{1}{(I-1)^2} (\vec{c}_I - \vec{\mu}_I) (\vec{c}_I - \vec{\mu}_I)^\top \\
 = & \hat{\underline{K}}_I + (I\vec{\mu}_I - I\vec{\mu}_I) [\dots]^\top + [\dots] (I\vec{\mu}_I - I\vec{\mu}_I)^\top + \underline{0} \\
 = & \hat{\underline{K}}_I + \underline{0} [\dots]^\top + [\dots] \underline{0}^\top + \underline{0} = \hat{\underline{K}}_I
 \end{aligned}$$

q.e.d.